

# Feature: Zernike & Co. - Description of Aberrations by Polynomials

## General

The described feature is available for all  $\mu$ Shape™ versions. Some versions possibly do not offer the whole complexity as described.

The  $\mu$ Shape™ Professional software determines the desired aberration (surface deviation resp. wave aberration) by detecting the phase value in each point (camera point) individually and results in a continuous data map without phase steps showing the deviation from the reference [G1]. In the case of a Mega-Pixel camera chip your sample is therefore described generally by about 800,000 independent data points. This huge number of data points can be reduced e.g. by describing the detected aberration by a polynomial approximation.  $\mu$ Shape™ offers two different sets of orthogonal polynomials: Zernike polynomials for circular apertures and Legendre polynomials for rectangular apertures. Describing the aberration by polynomials reduces the number of description parameters significantly and often makes it easier to compare different samples.

Polynomial presentation is available as an analysis option and as (sample) normal data definition (see [A1]). For export of polynomials see [G2].

## 1. Describing Aberrations by Polynomials

To describe an aberration by polynomials, a set of specific polynomials is fitted to the data map. Like any other fit, polynomials are not able to describe each aberration exactly. Commonly there remains a residual error which is not covered by the fit. Especially local defects are often ignored. You should always be aware of the residual error and should check it.

Using orthogonal polynomials assures that the selected polynomials always give the best description of the aberration and the coefficients do not change when adding further polynomials. In  $\mu$ Shape™ the required definition area (unit circle for Zernikes resp. rectangle for Legendres) is defined by setting an appropriate special analysis mask (see [G3] for more details).

### 1.1. Zernike Polynomials

Zernike polynomials


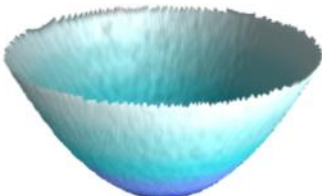
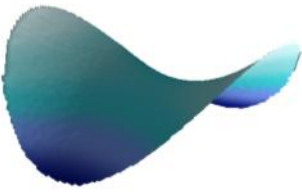
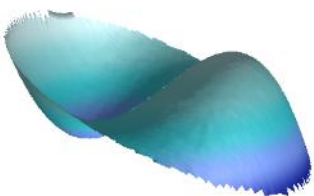
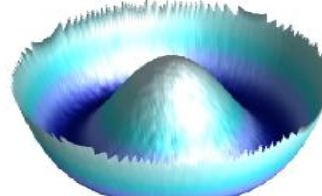

$$Z_n^m(r, \mathbf{j}) = R_n^m(r) \cdot (\cos(m\mathbf{f}) + \sin(m\mathbf{f}))$$

consist of a radial and an azimuthally term and define an orthogonal set of polynomials on a unit circle. The parameter  $n$  defines the maximal polynomial order, whereas  $m$  the minimal polynomial order and the multiple of the angle defines.

Zernike polynomials are widely spread used in optics especially to describe wave aberrations. So for example the low order Zernike polynomials can be used to describe imaging errors of optics by Seidel aberrations [see G4]. Two representations are commonly used: according ISO 10110 and as used by Max Born and Emil Wolf in their book "Principles of Optics". Both use of course the same polynomials but the identification of individual coefficients differs. In ISO they are sorted by the sum of the parameter  $m$  and  $n$ , whereas in Born&Wolf notation they are sorted according to the parameter  $n$ .

$\mu$ Shape™ calculates Zernike polynomials up to  $n_{max}=22$  and  $m_{max}=8$ . This includes polynomials up to 30<sup>th</sup> order. Up to 16<sup>th</sup> order the Zernike order is captured completely meaning all possible Zernike polynomials according ISO are available.

The following table shows the first lower order Zernike polynomials:

<p style="text-align: center;">tilt</p>  <p style="text-align: center;"><math>(C_1 \cdot \cos f + C_2 \cdot \sin f)r</math></p>	<p style="text-align: center;">defocus</p>  <p style="text-align: center;"><math>C_3 \cdot (2r^2 - 1)</math></p>	<p style="text-align: center;">astigmatism</p>  <p style="text-align: center;"><math>(C_4 \cdot \cos 2f + C_5 \cdot \sin 2f)r^2</math></p>
<p style="text-align: center;">coma</p>  <p style="text-align: center;"><math>(C_6 \cdot \cos f + C_7 \cdot \sin f)(3r^3 - 2r)</math></p>	<p style="text-align: center;">spherical aberration</p>  <p style="text-align: center;"><math>C_8 \cdot (6r^4 - 6r^2 + 1)</math></p>	<p style="text-align: center;">trifoil</p>  <p style="text-align: center;"><math>(C_9 \cdot \cos 3f + C_{10} \cdot \sin 3f)r^3</math></p>

### 1.2. Legendre Polynomials

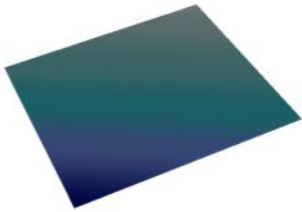
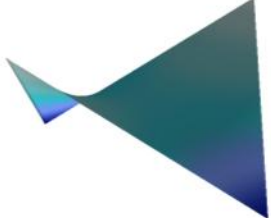

Legendre polynomials




$$L_{nm}(x, y) = P_n(x) \cdot P_m(y)$$

consist of two 1-dimensional polynomials, each creating an orthogonal system. They are defined on a unit rectangle. The parameter  $n$  and  $m$  define the maximal polynomial order of the corresponding polynomial. Legendre polynomials are used to describe aberrations of rectangular apertures.



$\mu$ Shape™ offers Legendre polynomials up to  $n_{max}=16$  and  $m_{max} = 16$ .

The following table shows the first lower order Legendre polynomials:

	 <p style="text-align: center;"><math>L_{11} \cdot xy</math></p>	 <p style="text-align: center;"><math>\frac{1}{2} \cdot L_{20}(3x^2 - 1)</math></p>
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$L_{10} \cdot x + L_{01} \cdot y$		
		
$\frac{1}{2}(L_{20}(3x^2 - 1) + L_{02}(3y^2 - 1))$	$\frac{1}{2} \cdot L_{21}(3x^2 - 1)y$	$\frac{1}{4} \cdot L_{22}(3x^2 - 1)(3y^2 - 1)$

## 2. Polynomial Analyses with $\mu$ Shape

The polynomial analyses are activated by selecting the desired analysis type from the *Analysis* menu or by pressing the corresponding button located in the Function toolbar  $f(x,y)$ , i.e.  or . A mask dialog opens to define the required circular or rectangular mask. It defines the fit area used for the polynomial fit. For a convenient use it is possible to link the mask to the analysis mask or – if the analysis mask is of appropriate type – to equalize both [see G5].

Basing on the valid aberration data inside the defined unit area (mask) the polynomial coefficients are calculated. The calculated coefficients are automatically displayed as table view.

Alternatively also graphical views can be created by pressing the corresponding button on the view toolbar. In this case the appropriate aberration map is determined using the calculated coefficient set. On the settings dialog of the corresponding polynomial analysis you can specify what shall be displayed as graphical view: the fit result on the actual aperture respectively on the unit area, or the residual map showing the part of the aberration which can't be described by the polynomial fit. The RMS-value of the residual can be seen as qualifying parameter for the fit.

All polynomial coefficients and appropriate analysis parameters are available for the parameter export [G2] and the Measurement Protocol [G6].

## 3. Reference Documents

- [A1] Add-on Module Sample Normal Data ("Addon\_SND.pdf")
- [G1] Feature: Phase Determination ("mShape\_PhaseDetermination.pdf")
- [G2] Feature: Storing measurement results ("mShape\_StoringResults.pdf")
- [G3] Feature: Masking ("mShape\_Masks.pdf")
- [G4] Feature: Analysis of Seidel aberrations ("mShape\_Seidel.pdf")
- [G5] Feature: Automatic Pupil Finding & Mask Coupling ("mShape\_AutoMasks.pdf")
- [G6] Feature: Printing Results ("mShape\_PrintingResults.pdf")

The references can be found in our download area ([www.trioptics-berlin.com/downloads](http://www.trioptics-berlin.com/downloads)).

If you need more information don't hesitate to contact us at [software@trioptics-berlin.com](mailto:software@trioptics-berlin.com).