Knowledge Base: Phase shifting

General

This document describes fundamental algorithms used for phase determination in µShape™ Professional.

A two-beam interferometer of e.g. Twyman-Green or Fizeau type determines the phase difference between two beams. The phase difference is equal to the optical path difference (OPD) between the reference and the corresponding sample beam. The phase difference can be seen as fringe pattern on a screen or camera sensor. Here each fringe represents the same phase value in the interference pattern like a height contour map. The difference between two neighbored fringes is a half of the used measurement wavelength. The most common method to determine the fringe order and to calculate a continuous wave front is phase-shifting, sometimes also called phase sampling.

1. Advantages and Disadvantages of Phase Shifting

In phase shifting interferometers several phase measurements with defined phase offsets are carried out and combined together to determine the final phase map. Such interferometers need a light source with a stable frequency over the whole phase shifting cycle. Furthermore a possibility is required allowing shifting the phase between the reference surface and the sample in known fractions of the wavelength. Mostly used are piezo-driven reference mirrors. Large-aperture systems sometimes use wavelength shifting of their light source. To avoid phase measurement errors a stable environment is needed until all phase steps are collected.

Using a phase shifting technique has the advantage that the sample area is detected automatically by the area where the intensity is modulated over the various phase steps. So it is not necessary to define a measurement area by setting a measurement mask, although it can be useful. Finally, phase-shifting methods deliver the phase value of each image pixel independently. They can deal with interrupted field of views (e.g. multiple apertures, apertures with holes inside) much better than methods based on overall phase descriptions such as Fourier transformations.

The disadvantages are the need of a phase-shifting device and stable measurement conditions for a certain time span, typically a few seconds least.

2. Phase Shifting Methods

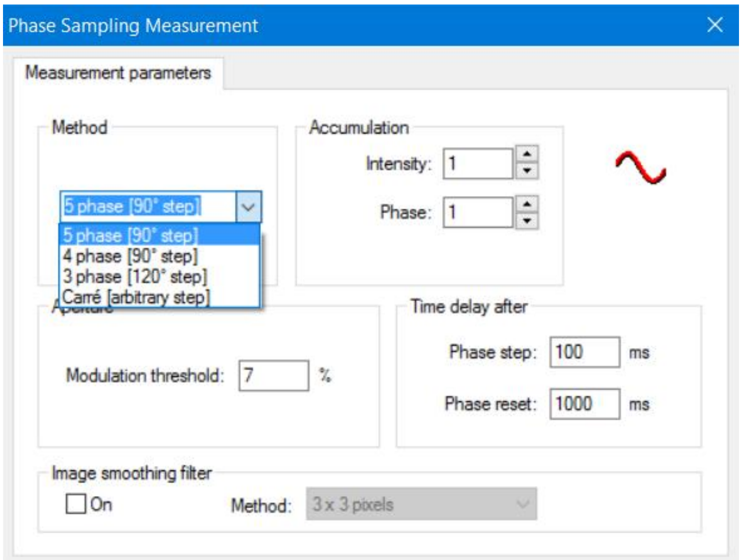
The aim of any phase shifting method is to determine the phase map $\Phi(x,y)$ from a set of detected intensity distribution $I(x,y)$ (fringe images) differing from each other by a known phase value. The relation between the intensity and the corresponding phase value is given by the following equation:

$$I(x,y) = I_0(x,y)\left[1 + V(x,y)\cos\left(\Phi(x,y) - \Phi_r\right)\right]$$

In this equation $I_0(x,y)$ is the mean intensity, $V(x,y)$ the visibility ("contrast") and Φ_r a reference phase.

As can be seen that the intensity depends on the phase difference $\left(\Phi(x,y) - \Phi_r\right)$. By changing the reference phase by different known values you get different intensity maps from which the phase map $\Phi(x,y)$ can be calculated. Changing the phase difference can be done e.g. by shifting the reference surface by specific fractions of the wavelength. The phase calculation is performed at each point (x,y) where modulation (change of intensity) is detected. It results in a wrapped phase map with values between 0 and 2π resp. – π and + π. This wrapped phase map contains phase gaps of 2π. These gaps are eliminated automatically by an algorithm called phase unwrapping. It considers the phase values of neighboring pixels to eliminate phase gaps larger than 2π. This unwrapped (continued) phase map is the initial result of the phase measurement and is saved in the measurement file (*.ms). All further analyses like smoothing, adjustment correction and additional analyses like e.g. Zernike are carried out on that raw phase data.

In the µShape™ software different types of phase shifting algorithms are implemented. They are selected and parameterized by the phase measurement dialog.



2.1. 4 Phase Algorithm

The 4 phase algorithm calculates the phase distribution $\Phi(x,y)$ from four fringe images I_1 to I_4 with reference phase values of Φ_r = 0, π/2, π and 3π/2, corresponding to a phase shift of 90°.

$$\Phi = \arctan\left(\frac{I_2 - I_4}{I_1 - I_3}\right)$$

2.2. 5 Phase Algorithm (Hariharan)

The 5 phase algorithm is an extended 4 phase algorithm. In addition to the four measurements of the 4 phase algorithm, a fifth measurement I_5 with a reference phase of Φ_r = 2π is taken. In this case the phase distribution $\Phi(x,y)$ is given by the equation

$$\Phi = \arctan\left(\frac{2(I_2 - I_4)}{I_1 - 2\cdot I_3 - I_5}\right)$$

2.3. Carré Algorithm

While the 4 and 5 phase algorithm requires a constant phase step of π/2 (90°) between each fringe image, the Carré phase algorithm expects any but constant phase step between the four fringe images I_1 to I_4. The phase distribution $\Phi(x,y)$ is given by the equation

$$\Phi = \arctan\left(\frac{\sqrt{[3(I_2 - I_3) - (I_1 - I_4)] \cdot [(I_1 - I_4) + (I_2 - I_3)]}}{(I_2 + I_3) - (I_1 + I_4)}\right)$$

2.4. 3 Phase Algorithm

The 3 Phase algorithm uses 3 phase steps shifted by 120° each. In µShape™ it is available as add-on module because it is useful for special interferometer setups only. That 3 phase algorithm calculates the phase distribution $\Phi(x,y)$ from three fringe images I_1 to I_3 with reference phase values of $\Phi_r = 0, 2\pi/3$ and $4\pi/3$:

$$\Phi = \arctan\left(\frac{\sqrt{3}(I_3 - I_2)}{2 \cdot I_1 - I_2 - I_3}\right)$$

2.5. Comparison

The most robust algorithm is the 5 phase algorithm and therefore it is the default one in the µShape™. It is immune against second order nonlinearities of the detector. That means it gives better error compensation.

The advantage of the Carré algorithm is that it also works with a total phase shift smaller than 2π.

3. Phase Step Map

Besides the phase calculation it is also possible to determine the mean phase step at each pixel. This phase step map is only calculated for the 5 phase algorithm and the Carré algorithm.

For 5 phase algorithm the mean phase shift for each pixel is calculated from the four intensity maps by the formula

$$d = \arccos\left(\frac{I_0 - I_4}{2(I_1 - I_3)}\right)$$

resp.

$$d = \arcsin\sqrt{\frac{3 - \frac{I_0 - I_3}{I_1 - I_2}}{3}}$$

Ideally, each pixel should show a value of π/2. If there are greater divergences from this value for a large number of pixels (RMS value>>0), it is recommend to carry out a new phase shifter calibration.

In case of Carré the mean phase step is given by

$$d = \arctan\sqrt{\frac{3(I_2 - I_3) - (I_1 - I_4)}{(I_1 - I_4) + (I_2 - I_3)}}$$

If the phase step map shows distortions or other effects, the phase shifting unit should be checked for wobbling or rolling.

4. Methods for Phase Shifter Calibration

For the most phase shifting methods the phase step must be known and equal for all phase steps of one measurement. For that purpose the phase-shifter has to be calibrated. In µShape software that is part of the calibration mode. Two methods are available for PZT based phase-shifter units: one for systems with linear relation between voltage and length change of the piezo and a second one for non-linear behavior.

4.1. Linear Phase Shifter Calibration

It is assumed, that the phase value ϕ changes linear with the applied voltage V:

$$j(V) = a_{Offset} + a_1 \cdot V$$

Beginning with the start values saved in the device.ini file, the five reference phases are set and the corresponding interferograms $I_0(x,y)$, $I_1(x,y)$, … $I_4(x,y)$ are detected. From these five interferograms the phase step ϕ at each point (x,y) can be calculated.

$$d = \arccos\left(\frac{I_0 - I_4}{2(I_1 - I_3)}\right)$$

resp.

$$d = 2\arcsin\sqrt{\frac{3 - \frac{I_0 - I_3}{I_1 - I_2}}{4}} \qquad \text{for } I_1 = I_3$$

If δ differs from π/2 more than a specified tolerance the coefficient a_1 is modified and another set of five interferograms is analyzed. The whole process is repeated iteratively until δ=π/2 (± tolerance) or the given iteration steps are reached.

4.2. Non-linear Phase Shifter Calibration

In this case it is assumed, that the dependency of the phase value ϕ from the voltage V is given by

$$j(V) = a_{Offset} + a_1 \cdot V + a_2 \cdot V^2 + a_3 \cdot V^3$$

For calibration an intensity scan over the whole voltage range at nine points is done. For each observed point the sinus lapse of the detected intensity is analyzed.

First the scan is normalized to the maximum/minimum value. Then the phase value ϕ(V) is calculated by applying the arcsine function. This phase map is unwrapped and a least squares fit is applied. Finally the correlation factor is computed. The point with the greatest correlation is used as result.

If you need more information don't hesitate to contact us at software@trioptics-berlin.com.